

# Forecasting Volatility in Commodity Markets

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Methods for forecasting long-run commodity price volatility that combine time-series modeling with market expectations derived from options prices outperform forecast methods using only market expectations or only time series.



## Summary findings

Commodity prices have historically been among the most volatile of international prices. Measured volatility (the standard deviation of price changes) has not been below 15 percent and at times has been more than 50 percent. Often, the volatility of commodity prices has exceeded that of exchange rates and interest rates.

The large price variations are caused by disturbances in demand and supply. Stockholding leads to some price smoothing, but when stocks are low, prices can jump sharply. As a result, commodity price series are not stationary and in some periods they jump abruptly to high levels or fall precipitously to low levels relative to their long-run average. Thus it is difficult to determine long-term price trends and the underlying distribution of prices.

The volatility of commodity prices makes price forecasting difficult. Indeed, realized prices often deviate greatly from forecasted prices, which has led to the practice of giving forecasts probability ranges. But assigning probability ranges requires forecasting future price volatility, which, given uncertainties about true price distribution, is difficult.

One potentially useful source of information for forecasting volatility is the volatility forecasts imbedded in the prices of options written on commodities traded in exchanges. Options give the holder the right to buy (call) or sell (put) a certain

commodity at a certain date at a fixed (exercise) price. Options prices depend on several variables, one of which is the expected volatility up to the maturity date. Given a specific theoretical model, the market prices of options can be used to derive the market's expectations about price volatility and the price distribution.

Kroner, Kneafsey, and Claessens systematically analyze different methods' abilities to forecast commodity price volatility (for several commodities). They collected the daily prices of commodity options and other variables for seven commodities (cocoa, corn, cotton, gold, silver, sugar, and wheat). They extracted the volatility forecasts implicit in options prices using several techniques. They compared several volatility forecasting methods, divided into three categories:

- (1) Forecasts using only expectations derived from options prices.
- (2) Forecasts using only time-series modeling.
- (3) Forecasts that combine market expectations and time-series modeling (a new method devised for this purpose).

They find that the volatility forecasts produced by method 3 outperform the first two as well as the naive forecast based on historical volatility. This result holds both in and out of sample for almost all commodities considered.

This paper — a product of the Debt and International Finance Division, International Economics Department — is part of a larger effort in the department to study the short- and long-run behavior of primary commodity prices and the implications for developing countries of movements in these prices. The study was funded by the Bank's Research Support Budget under the research project "Measurement of Commodity Price Volatility" (RPO 676-73). Copies of this paper are available free from the World Bank, 1818 H Street NW, Washington, DC 20433. Please contact Faten Hatab, room H8-099, extension 35835 (22 pages). November 1993.

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## **Forecasting Volatility in Commodity Markets**

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## Summary

Commodity prices have historically been one of the most volatile of international prices. Measured volatility (the standard deviation of price changes) has not been below 15% and at times has been more than 50%. During many periods the volatility of commodity prices has exceeded the volatility in exchange rates and interest rates. The cause of the large price variations are disturbances in demand or supply. Stockholding leads to some price smoothing but when stocks are low, prices can jump very sharply. As a result, commodity price series have two important features. First, they display non-stationarity. Second, there are periods when prices jump abruptly to very high levels (or fall to very low levels) relative to their long run average. These two features make it very difficult to determine the long-term price trend and the nature of the underlying distribution of prices.

The high levels of commodity price volatility make price forecasting extremely difficult and, indeed, large ex-post deviations of realized prices from forecasted prices are common. This has motivated the practice of providing probability ranges around the forecast. This, in turn, requires forecasts of future price volatility which, given the uncertainty about the nature of the true price distribution, has also proven difficult. One potentially useful source is the volatility forecasts imbedded in prices of options written on commodity prices traded on exchanges. Options give the holder the right to buy (call) or sell (put) a certain commodity at a certain date at a fixed (exercise) price. Option prices depend on a number of variables, one of which is the expected volatility over the horizon to the maturity date. Thus, given a specific theoretical model, market prices of options can be used to derive the market's expectation of price volatility and the price distribution.

This paper provides a systematic analysis--covering several commodities--of the ability of different methods of forecasting commodity price volatility. Daily prices of commodity options and other variables were collected for seven commodities (cocoa, corn, cotton, gold, silver, sugar and wheat). The volatility forecasts implicit in the option prices were then extracted using several techniques. The paper compares several different forecasts of commodity price volatility, divided into three categories: (1) forecasts using only expectations derived from options prices; (2) forecasts using only time series modelling; and (3) forecasts which combine market expectations and time series modelling. Methods (1) and (2) have been used extensively in the literature, while use of method (3) for this purpose is new. The paper finds that combining market expectations and time series modelling (method (3)) gives volatility forecasts which outperform both market expectations forecasts and time series forecasts as well as the naive forecast based on historical volatility. This result holds both in and out of sample for virtually all commodities considered. These results hold promise for deriving probability distributions for commodity price forecasts which will perform considerably better than methods now being used.

## I. Introduction.

Commodity prices have historically been one of the most volatile of international asset prices. Over the period 1972-1990, for example, the volatility of non-oil commodity prices in nominal terms, as measured by the annualized standard deviation of percentage changes during the previous 24 months, was not below 15% and peaked at more than 50% in 1975. It is not surprising, then, that efforts to forecast commodity prices have been largely unsuccessful. In fact, it can be expected that ex-post forecast errors will continue to display a large distribution around zero, whether the forecasts are made using futures prices or specialists' assessments. For this reason, it is important to construct a measure of confidence in the price forecast. The traditional way to express confidence in a forecast is to bound it by a confidence interval, that is, to give an interval forecast and an associated probability. This is easily done in a world where volatility is constant, but if volatility is itself changing, then a forecast of volatility must be derived before an interval forecast can be constructed. Volatility forecasts are also crucial in the pricing of options contracts because all else equal, a higher forecasted volatility should result in higher options prices. So an investor with a better forecast of volatility than the market's should be able to exploit the forecast to make excess returns.

The purpose of this paper is to develop methods of forecasting commodity price volatility over long time horizons, here taken as 225 days. Several methods of forecasting volatility over short horizons already exist in the literature, but many of these methods do not carry over to long-horizon forecasts. For example, implied standard deviation forecasts derived from options prices as in Latane and Rendelman (1976), Beckers (1981), Wei and Frankel (1991), and many others, may be appropriate for short-term forecasts, but are unreliable for long-term forecasts because trading is very thin in options that are far from their expiration dates. Also, traditional time series methods, like ARMA models on moving standard deviations as in Cao and Tsay (1991), or ARMA models on bid-ask spreads or daily price ranges, as in Taylor (1987), are not likely to give useful forecasts because a 225-day forecast will generally be simply the unconditional mean of the series. To illustrate, the 225-day forecast from the ARMA(1,1) model

$$y_t = \omega + \theta y_{t-1} + \phi \epsilon_{t-1} + \epsilon_t$$

is

$$\hat{y}_{t,t+225} = \omega \left( \sum_{i=0}^{224} \theta^i \right) + \theta^{225} y_t + \theta^{224} \phi \epsilon_t,$$

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We would like to thank seminar participants at the World Bank for their useful suggestions. All errors are ours alone.

which is approximately  $\omega/(1 - \theta)$  if  $|\theta| < 1$ . We develop a forecasting model which combines investors' forecasts with time series forecasts, and produces forecasts of long-term volatility which are more accurate than the short-term forecasting methods. This result holds both in-sample and out-of-sample for almost all of the commodities we consider, suggesting that our proposed forecasting model can be an effective tool for constructing interval forecasts and for the pricing of long-term options. The paper is organized as follows: Section II discusses some short-term volatility forecasting methods and Section III presents our new forecasting framework. Section IV discusses the data, Section V presents the results, and Section VI concludes.

## II. Existing Forecasts.

**II.1. Implied Standard Deviations.** One popular method of forecasting volatility uses option prices to measure investors' expectations of future volatility. An option is a contract which permits, but does not require, the holder to buy (sell) the underlying asset at a predetermined price. Clearly, the more volatile the price of the underlying asset, the more likely it is that the option will have value, consequently the higher the option's price. If the option market is efficient, then investors' expectations of future volatility as embodied in option prices should be the best volatility forecast available. More specifically, option prices are functions of four observable variables (the price of the underlying asset, the exercise price of the option, the time to maturity of the option, and the risk-free rate of interest) and one unobservable variable (the expected volatility of returns on the underlying asset over the life of the option). Since the option price is itself observable, and since the option price is a monotonically increasing function of expected volatility, one can use an option pricing model to back out the market's expectation of volatility over the remaining life of the option (Latane and Rendleman, 1976). This forecast of volatility is often called the implied standard deviation, or ISD. We refer to it as a "market-based" forecast because it is based entirely on the expectations of participants in the options market (given a particular option pricing model).

To compute the ISD we require an option pricing formula for commodity futures options. Most options on commodities are American options, meaning that they can be exercised either at any date on or before maturity or at any time within a specific period (e.g., one month) before maturity<sup>1</sup>. This early exercise feature of commodity options means that they should sell at a

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<sup>1</sup> Early exercise is important when dealing with options on futures contracts. Ignoring storage costs and convenience yields, the futures price is  $F(T) = S_0 e^{-rT}$ , so the futures price declines to the spot price as expiration nears. This makes the behavior of futures prices similar to that of a stock price that pays out continuous dividends. These "implicit dividends" can only be reaped if the option holder exercises the option.

premium relative to European options, which allow the option holder to exercise the option only on the maturity date of the option. This in turn means that the use of the standard European option pricing formula of Black (1976) will result in ISD's which are overstated. The positive bias in the ISD occurs because the European formula assumes that the higher option price is due to higher volatility (and therefore a higher estimated ISD is obtained), when in reality the higher price simply reflects the early exercise premium<sup>2</sup>. Unfortunately, no closed-form solution exists for American options, though several approximation methods exist. We adopt the method of Barone-Adesi and Whaley (1987), who provide an efficient approximation for pricing American options on commodity futures.

Suppose that the underlying futures price follows the stochastic differential equation

$$(1) \quad dF/F = \zeta dt + \sigma dz,$$

where  $F$  is the commodity futures price,  $\zeta$  is the instantaneous expected relative price change of the commodity,  $\sigma$  is the instantaneous standard deviation,  $t$  is time and  $z$  is a Wiener process. Then if the interest rate  $r$  is constant and no arbitrage opportunities exist, Black (1976) shows that the price of a commodity futures option,  $C$ , must follow the partial differential equation

$$(2) \quad \frac{1}{2}\sigma^2 F^2 C_{FF} - rC + C_t = 0$$

where subscripts represent partial derivatives of the variable with respect to the subscript. For a European option with no early exercise privilege, the boundary condition requires that the maturity value of the option be equal to  $\max\{0, F_T - X\}$ , where  $F_T$  is the futures price at maturity and  $X$  is the exercise price. This boundary condition is applied to (2) to get Black's European commodity futures option pricing formula

$$(3) \quad c(F_t, T, r, X, \sigma) = e^{-rT} [F_t N(d_1) - X N(d_2)],$$

where  $T$  is time to maturity,  $N(\cdot)$  is the cumulative normal distribution, and

$$d_1 = [\ln(F_t/X) + \frac{1}{2}T\sigma^2]/\sigma\sqrt{T}$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

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<sup>2</sup> To demonstrate the potential magnitude of the bias, consider the May/1985 soybean futures option selling on November 2, 1984 with an exercise price of \$600. The futures price was \$664.75, the spot price was \$627.50, the call price was \$76.00, and the annualized interest rate was 9.737%. Using Black's (1976) formula for pricing European futures options the ISD is 0.2272, while using the Barone-Adesi and Whaley American option pricing approximation (which will be discussed shortly), the ISD is 0.2174. The overstatement using the European formula is thus about 5%.

However, when early exercise is possible, the American option boundary conditions must be used, and a closed form solution no longer exists. Barone-Adesi and Whaley (1987) propose an approximate solution to this problem. Without going into the details of their derivation, define

$$\begin{aligned} A_2 &= (F^*/q_2)\{1 - e^{-rT}N(d_1^*)\} \\ d_1^* &= [\ln(F^*/X) + \frac{1}{2}\sigma^2T]/\sigma\sqrt{T} \\ q_2 &= [1 + \sqrt{1 + 8r/(\sigma^2K)}]/2 \\ K &= 1 - e^{-rT}, \end{aligned}$$

where  $F^*$  is the futures price that satisfies

$$F^* - X = c(F^*, T, r, X, \sigma) + \{1 - e^{-rT}N(d_1^*)\}F^*/q_2$$

and  $c(\cdot)$  is Black's theoretical call price in equation (3) above. Then the approximate formula for the price of an American commodity futures call option at time  $t$ ,  $C(F_t, T, r, X, \sigma)$ , is

$$(4) \quad C(F_t, T, r, X, \sigma) = \begin{cases} c(F_t, T, r, X, \sigma) + A_2(F_t/F^*)^{q_2} & \text{if } F_t < F^* \\ F_t - X & \text{if } F_t \geq F^* \end{cases}$$

Three important observations merit mention. First, while this formula is mathematically complicated, it is easily programmed. Second, Barone-Adesi and Whaley (1987) show that this approximation becomes exact as the time to expiration gets large. In this paper we are focusing on long-horizon forecasts, so the approximation error should be quite small. And third, notice that there are only six variables in this formula —  $C$ ,  $X$ ,  $F$ ,  $r$ ,  $T$  and  $\sigma$ . The first five are observable and therefore can be used to solve for  $\sigma$ . This  $\sigma$ , the ISD, is the measure of volatility implied by the option price, that is, the market's expectation of volatility over the remaining life of the contract.

In this paper we will evaluate a set of forecasts based on ISD's. But for any given day in our data set and for any given option maturity, there are several different options traded, one for each exercise price. For example, on Nov 2, 1988, there were 20 wheat contracts which expired in March 1989, each with a different strike price. Therefore, we can extract 20 different ISD's, or twenty different forecasts of volatility between Nov/88 and Mar/89<sup>3</sup>, one from each contract. We used three different methods to collapse these multiple ISD forecasts into a single volatility forecast. The first takes the ISD from the contract for which the price of the option is most sensitive

<sup>3</sup> All 20 of these forecasts do not return the same ISD. The fact that they do not is an indication of one or more of the following: option market inefficiencies; the impact of price discreteness in an option pricing model that assumes continuous prices; and/or misspecification in the option pricing model.



to changes in the volatility of the underlying commodity. Usually, this is the at-the-money option, but occasionally it is the near-the-money option<sup>4</sup>. This contract is used to extract the ISD for three reasons. First, because this contract price is sensitive to volatility, it should therefore return the most accurate measure of volatility. Second, for at-the-money options on futures contracts, the value added by the American feature is the smallest (Ramaswamy and Sundaresan, 1985). While we correct for the American feature, the correction is not perfect and we prefer to use the ISDs that are least influenced. Third, at-the-money options have the smallest bias when volatility is not constant. The Black model is (approximately) linear in volatility for at-the-money options, which implies that the at-the-money implied volatility estimates will result in only a small bias when volatility is stochastic (Hull and White, 1987).

So our first set of forecasts is the ISD extracted from the contract with the highest derivative of the call price with respect to volatility, in other words the  $\sigma$  which solves the equation

$$(5) \quad C(F_t, T, r, X_i, \sigma) - C^*(t, T, X_i) = 0$$

where  $C^*(t, T, X_i)$  is the observed call price at time  $t$  for a contract with time to expiration  $T$  and exercise price  $X_i$ ,  $C(\cdot)$  is the Barone-Adesi and Whaley (1987) formula given in (4) above, and  $i$  is chosen to maximize  $\frac{\partial C}{\partial \sigma}$ . Throughout this paper, we refer to this  $T$ -period forecast of volatility at time  $t$  as ISDAT <sub>$t, T$</sub> .

Using ISDAT, however, ignores information about volatility that is available in other contracts. We therefore propose two other market-based forecasts of volatility which do not ignore the potentially useful information in the prices of options which are not at-the-money. First, we use a weighted average of all the ISD's that can be computed on a given date  $t$  for a given time to expiration  $T$ , with the weights being the derivatives of the option price with respect to volatility. More specifically, the forecast we use is

$$(6) \quad \text{ISDAVG}_{t, T} = \frac{\sum X_i \gamma_{TX_i} \sigma_{TX_i}}{\sum X_i \gamma_{TX_i}}$$

where  $\sigma_{TX_i}$  is the ISD from a call option with  $T$  days to expiration and with exercise price  $X_i$ , and  $\gamma_{TX_i}$  is the derivative of the price of this call option with respect to volatility. We chose this weighting mechanism because options which are far away from the money have prices which are

<sup>4</sup> "At-the-money" refers to the option whose exercise price is closest to the spot price of the underlying asset. The difference in the ISD's from the at-the-money option and the near-the-money option are likely to be very small. To illustrate, in previous work by the authors using soybean data, the correlation between at-the-money ISD's and just-in-the-money ISD's from contracts with greater than 260 days to maturity is 0.9891.

not as sensitive to volatility, and should therefore get less weight in our forecast than the more sensitive, closer-to-the-money options. We call this forecast ISDAVG, because it is a weighted average of ISD's.

The second method we use to account for all the information in the options market chooses the single measure of volatility which most closely approximates the observed pattern of ISD's across different strike prices. More specifically,

$$(7) \quad \text{ISD}_{1,t,T} = \arg \min_{\sigma} \sum_{X_i} \gamma_{TX_i} [C^*(t, T, X_i) - C(F_t, T, r, X_i, \sigma)]^2,$$

where all variables are as defined above. This method recognizes that the true volatility is the same for all the options on a given futures contract, regardless of the exercise price, and chooses the single estimate of volatility which is closest to satisfying the option pricing equation for all exercise prices. "Closeness" is measured by the mean squared deviation between the observed and theoretical prices, aggregated over all contracts with a given maturity and weighted by  $\gamma_{TX_i}$ . We call this forecast ISD1 because it uses all the information to extract one estimate of the ISD.

To summarize, three different forecasts of volatility are extracted from market expectations using the ISD's from the option pricing approximation of Barone-Adesi and Whaley (1987). For each day of our data set, we calculate these three forecasts for each contract expiration. For example, using wheat futures options, on November 2, 1988 there were four different expirations being traded (Dec/88, Mar/89, May/89 and Jul/89), meaning that we have three different forecasts of volatility for each of four different horizons. The ISDs from the contract which is closest to 225 days from expiration are used as our market-based forecasts.

Several problems are inherent in these market-based forecasts, however. Perhaps most importantly, the trading of options with maturities over six months is often so thin that long horizon forecasts of volatility using ISDs are potentially unreliable. Also, most option pricing models assume that volatility is constant, so when forecasts are extracted from these models in a world of dynamic volatility, it is not clear what is really being forecast. Finally, it is possible that the options market is not efficient and/or the option pricing formulas are incorrect, as evidenced by the different ISDs that are extracted from different exercise prices (see footnote 3). These problems suggest that the forecasts extracted from option pricing models might not be the best available forecasts.

**II.2. Time Series Forecasting.** Another method that has been proposed to forecast volatil-

ity involves time series modelling of the variances (see, for example, Engle and Bollerslev, 1986). Many assets are characterized by time varying variance, and consequently require dynamic models of volatility. One set of models which has become popular in finance is the Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized ARCH (GARCH) models of Engle (1982) and Bollerslev (1986), in which variances are modeled as an ARMA process. The popularity of GARCH models stems from their ability to capture volatility clustering, a feature common to financial time series. See Bollerslev, Chou and Kroner (1992) for a survey of GARCH applications in financial modelling.

If  $S_t$  is the commodity price at time  $t$  and  $\mathfrak{I}_{t-1}$  is the information set at time  $t - 1$ , then a simple GARCH(1,1) model is<sup>5</sup>

$$\begin{aligned} \ln S_t - \ln S_{t-1} &= \mu + \epsilon_t \\ \epsilon_t | \mathfrak{I}_{t-1} &\sim N(0, h_t) \\ (8) \quad h_t &= \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1}. \end{aligned}$$

Here,  $h_t$  is conditional variance of returns. Given an initial value for  $h_t$  and parameter estimates for  $\omega$ ,  $\alpha$  and  $\beta$ , equation (8) can be used to forecast volatility at any given horizon. The forecasting equation is simply (Engle and Bollerslev, 1986, equation 22)

$$E(h_{t+s} | \mathfrak{I}_t) = \begin{cases} \omega + \alpha \epsilon_t + \beta h_t & \text{if } s = 1 \\ \omega + (\alpha + \beta) E(h_{t+s-1} | \mathfrak{I}_t) & \text{if } s \geq 2, \end{cases}$$

or, using recursive substitution,

$$(9) \quad E(h_{t+s} | \mathfrak{I}_t) = \begin{cases} \omega + \alpha \epsilon_t + \beta h_t & \text{if } s = 1 \\ \omega[1 + (\alpha + \beta) + \dots + (\alpha + \beta)^{s-2}] + (\alpha + \beta)^{s-1}(\omega + \alpha \epsilon_t + \beta h_t) & \text{if } s \geq 2. \end{cases}$$

Using equation (9) to forecast conditional variance at horizons 1, 2, ...,  $T$  permits us to obtain a forecast of the variance over the  $T$ -period horizon by simply summing the individual forecasts. So this volatility forecast, which we call GARCH, is the square root of the aggregated forecasted variances,

$$(10) \quad \text{GARCH}_{t,T} = \sqrt{\sum_{i=1}^T E(h_{t+i} | \mathfrak{I}_t)}.$$

Akigray (1989), Lamoureux and LastRAPES (1991), and Day and Lewis (1992) demonstrate the usefulness of the GARCH model in developing short-run volatility forecasts in various equity markets.

<sup>5</sup> The simple mean equation in the GARCH model reflects the fact that the volatility measure of interest is a measurement of returns volatility. Therefore the dynamics in the mean equation are not modelled.

A second time series-based forecast of volatility, which is based only on historical returns, is included in our comparisons as a simple benchmark that at a minimum more complex models must beat. This forecast, which we call  $HIST_{t,T}$  (for historical), is simply the sample standard deviation of returns over the previous 7 weeks. This forecast is included because Bartunek and Mustafa (1991) find that for the stocks they used, it outperformed both ISD-based forecasts and GARCH-based forecasts for long horizons (80 and 120 days). However, this result seems to contradict much of the extant literature.

To summarize, in addition to the three ISD-based forecasts, we also have two time series-based forecasts of volatility. The first pure time series method uses a GARCH model to forecast volatility over the remaining life of the contract and the second time series method, HIST, uses the sample variance of returns over the previous seven weeks as a forecast of future volatility. But pure time series models, by definition, ignore the market's expectations of the future volatility and rely solely on the information contained in the past data. Consequently, we propose a third class of volatility forecasts which incorporates both time series analysis and market expectations of volatility.

### III. Combined Models.

Until recently, it was widely believed that the best forecasts of volatility came from the ISD models, because they could be expected to dominate any time series model that could be constructed. With the introduction of GARCH modelling, however, researchers are beginning to conclude that GARCH forecasts outperform ISD forecasts. See, for example, Bartunek and Mustafa (1991), Lamoureux and Lastrapes (1991) and Day and Lewis (1992). But the evidence in these papers and elsewhere also seems to indicate that while GARCH provides the best forecasts, ISD's still can be used to explain some of the forecasting error from the GARCH forecast. Intuitively this seems plausible since the GARCH forecast is conditional only on past information, while the ISD is a measure of market expectations regarding the future volatility and is conceivably constructed from a larger, more current information set. For this reason, we introduce the following forecasting model, which combines the GARCH-based model with the ISD-based model:

$$\begin{aligned}
 \ln S_t - \ln S_{t-1} &= \mu + \epsilon_t \\
 \epsilon_t | \mathcal{F}_{t-1} &\sim N(0, h_t) \\
 h_t &= \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} + \delta \sigma_{t-1}^2.
 \end{aligned}
 \tag{11}$$

In this model,  $\sigma_{t-1}$  is the at-the-money ISD from the option having closest to 100 days to expiration.

extracted from equation (5). If the 100-day ISD implies that early exercise is optimal, then we used the 100-day ISD from the previous day.<sup>6</sup> Models like this have been estimated before (see, for example, Day and Lewis, 1992), but only in the context of tests for market efficiency. They have not been used in forecasting exercises. The idea behind the market efficiency test is that if the options market is efficient then the ISD backed out from a properly specified options pricing model should capture all of the volatility of the spot prices that can be predicted based on the current information set. The implication is that all of the coefficients in the variance equation of model (11) should be zero except for the coefficient on the ISD term. If either  $\alpha$  or  $\beta$  remains significant upon inclusion of the ISD term, then there is information in the past time series of volatility which is not incorporated in the market's expectations of future volatility, but is relevant in predicting future volatility. This implies that the options are mispriced and past volatility data can be used to take advantage of the mispricing.

Forecasts from this model are made with the following equation:

$$(12) \quad E(h_{t+s}|\mathfrak{F}_t) = \begin{cases} \omega + \alpha\epsilon_t + \beta h_t + \delta\sigma_t^2 & \text{if } s = 1 \\ \omega + \delta\sigma_t^2 + (\alpha + \beta)E(h_{t+s-1}|\mathfrak{F}_t) & \text{if } s \geq 2, \end{cases}$$

Again, we forecast  $h_{t+s}$  for each period between now and our forecast horizon, and the square root of the sum of these forecasts is our forecast of volatility:

$$(13) \quad \text{COMB}_{t,T} = \sqrt{\sum_{i=1}^T E(h_{t+i}|\mathfrak{F}_t)},$$

where  $E(h_{t+i}|\mathfrak{F}_t)$  is computed from equation (12). We call this forecast  $\text{COMB}_{t,T}$  because it combines the ISD and GARCH forecasts. This gives us six forecasts of volatility: 3 ISD forecasts, 2 time series forecasts, and a combination ISD and GARCH forecast.

#### IV. Data.

The forecasting methods presented above are evaluated using daily data for cocoa, corn, cotton, gold, silver, sugar and wheat. The time span covered for each commodity varies slightly, depending on data availability, but usually extends from about January 1987 – November 1990. The ISD-based forecasts require data on futures prices, interest rates and options prices. The futures data is daily closing prices, obtained from Knight-Ridder Financial Services. The interest rates we use are the

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<sup>6</sup> We found that the horizon of the ISDs did not matter in equation (11). We therefore used 100-day ISDs instead of 225-day ISDs because they are much more heavily traded and are much more commonly analyzed in the literature.

treasury bill rates from the bill which expires closest to the time the option expires, as provided by Data Resources, Inc. The options price data for corn and wheat is daily closing prices, obtained from the Chicago Board of Trade, while the rest of the options data is daily closing prices, obtained from Data Resources, Inc. Information regarding the features of various options and futures contracts (such as the last trading day, contract months, etc.) is taken from the descriptions published by the different commodity exchanges. Table 1 provides a brief overview of the options data by commodity and serves to illustrate the magnitude of the data sets and the breadth of contracts traded per day.

TABLE 1

The  $GARCH_{t,T}$  and  $HIST_{t,T}$  forecasts require spot price data, which was obtained from Data Resources, Inc. In order to evaluate the forecasts, a measure of the "true" 225-day volatility is needed. One measure used in the literature and the one which we use here, is the realized standard deviation of returns over the forecast horizon. This is computed by calculating the square root of the average daily squared return over the forecast horizon. This is called ACTUAL to represent the actual volatility of returns over the period of interest. Clearly, comparing the various long-run forecasts to ACTUAL requires spot data which extends 225 days beyond the last option data, so the time span covered by our spot data is from about January 1987 – July 1991.

## V. Estimation and Results.

The three ISD-based 225-day volatility forecasts are computed from equations (5), (6) and (7) above. In order to construct the GARCH forecast and the COMB forecast, we need to estimate the GARCH model (equation 8) and the COMB model (equation 11). The relevant maximum likelihood estimates are presented in Table 2, with asymptotic  $t$ -statistics in parentheses<sup>7</sup>. The  $Q^2$  statistic, which tests for remaining serial correlation in the standardized squared residuals and is distributed  $\chi^2_{12}$  under the null of no remaining serial correlation, indicates that the estimated models adequately capture the dynamics in the second moments. One result of interest is that for many of the commodities (cotton and wheat are the exceptions), the variance equation coefficients sum to approximately one (i.e.  $\alpha + \beta \approx 1$ ). This means that shocks to the variance are persistent, i.e., shocks remain important determinants of the variance forecasts long after the shocks occur. This is easily seen by setting  $\alpha + \beta = 1$  in the second line of the GARCH forecasting equation (9).

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<sup>7</sup> The observation period used in estimating these models excluded the final eight weeks of our sample (40 observations) in order to facilitate out-of-sample forecasting later in this paper.

giving

$$E(h_{t+s}|\mathfrak{F}_t) = \omega(s-1) + h_{t+1}.$$

In this case the optimal variance forecast is simply the forecast of tomorrow's variance, adjusted for a drift component. This is important for our application because it implies that the long-term forecast from the GARCH models will not revert to the unconditional variance, as is common in long-run forecasts from ARMA models. Another result of interest is that the ISD in the combined model is highly significant for all commodities examined, suggesting that market expectations can help to predict variances. Also, the GARCH parameters ( $\alpha$  and  $\beta$ ) tend to drop in significance meaning that the ISD's capture much of the same information that GARCH does. This drop is most noticable in  $\beta$ . But the GARCH parameters tend to remain significant, suggesting a violation of options market efficiency. In other words, the ISD's contain information about future volatility that is not captured by the GARCH model, and the time series of volatility contains information about future volatility that is not incorporated in the option price. This suggests that the COMB model, which puts these two kinds of information into the same model, has potential to be a successful forecasting model.

TABLE 2

For each commodity, the six different forecasts and the actual variance over a 225-day forecast horizon are summarized in Table 3. Each block of the table presents summary statistics (average, minimum, maximum and number of observations) for the actual and forecasted variance for each of the different commodities studied<sup>8</sup>. Also, to illustrate the relationships among the various forecasts, Table 4 presents the correlation matrix for the corn forecasts. One observation from Table 3 is that the ISD-based forecasts tend to overstate true volatility (except for cocoa and wheat). There are several explanations for this, two of which are stochastic interest rates and stochastic volatility. For example, if the interest rate is stochastic then the ISD will capture both asset price volatility and interest rate volatility, so the ISD will be overstated. However, Ramaswamy and Sunderesan (1985) show that using the actual term structure at each point in time (as we do) eliminates much of the mispricing due to stochastic interest rates. Another observation is that the HIST forecast seems to be very accurate, on average. This is surprising, given that it was constructed as the sample

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<sup>8</sup> In this table and the two which follow, the samples from which the statistics are computed do not include the final eight weeks of data, which were withheld for out-of-sample comparisons. This explains part of the difference between the number of observations listed in Table 3 and the number of days listed in Table 1. The remaining difference is caused by withholding the first 34 observations in order to enable computation of the HIST forecasts. Also, the ISD-based forecasts have different numbers of observations because observations were dropped if the extracted ISD implied that early exercise was optimal.

standard deviation over moving 7-week windows, and the actual volatility is the sample standard deviation over the subsequent 225-day window. However, as we will see below, having an average forecast error close to zero does not make it a good forecast. The second most accurate forecasts, on average, are the COMB forecasts, suggesting that combining market expectations with time series methodology might improve forecasting ability. Another observation is that for some commodities (cotton, gold and perhaps wheat), the range of the GARCH forecasts is small, suggesting that the GARCH forecasts are almost constant. This is to be expected for long-horizon forecasts if  $\alpha + \beta$  is much less than one (see equation 9), as is the case for cotton and wheat. Finally, we see from Table 4 that the correlations among the three ISD forecasts are very high, and all correlations with HIST are low. We should therefore not be surprised if the ISD forecasts all perform similarly, while the HIST forecasts perform badly.

#### TABLES 3 AND 4

Many of these observations are evident in Figure 1, which presents the 225-day ISDAT, HIST, GARCH and COMB forecasts for corn along with the actual 225-day volatility.<sup>9</sup> For example, it is clear that ISDAT tends to overstate volatility and that the GARCH forecasts are relatively stable. The HIST forecasts seem to have little relationship to the actual variance being forecast, even though on average they might be close to the actual variance. The COMB forecasts seem to track the true volatility quite well, though the swings in the COMB forecast are much bigger than the swings in the realized volatility.

#### FIGURE 1

We use mean squared forecast errors (MSFEs) to formally evaluate each of the forecasts. Other metrics, like mean absolute forecast errors, gave virtually identical conclusions. Table 5 shows the mean squared forecast errors for each of the forecasting models for each commodity. It is clear from the table that, with the exception of silver, the GARCH and COMB models dominate all of the ISD forecasts and the historical volatility forecasts. GARCH in particular forecasts well, having the smallest MSFE for 4 of the 7 commodities and performing second best in two other cases. COMB has the smallest MSFE for two of the commodities, and has the second smallest MSFE for three others. Also, as anticipated from Table 4, the ISD forecasts all perform similarly, and the HIST forecasts tend to perform the worst.

#### TABLE 5

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<sup>9</sup> We do not present graphs of ISDAVG and ISD1 because they are very similar to the ISDAT graph.



Though the results are not reported here in order to conserve space, one interesting result is that the ISD's from options on futures contracts relatively near maturity (30 - 50 days to maturity) provide long-run volatility forecasts that are similar in accuracy to those provided by options on futures contracts that are far from maturity (225 days to maturity). It seems that there are two offsetting effects here. The first effect is that the near maturity options are more heavily traded than the distant options and consequently are priced more precisely, implying a more accurate ISD. But this effect is countered by the fact that the ISD from the near to maturity option must be extrapolated to span the desired horizon, thus reducing its accuracy. In contrast, using distant horizon contracts eliminates the need for extrapolating, at the cost of using infrequently traded options.

But the true test of a model's ability to forecast can only be accomplished through out of sample forecasting. Therefore, the 225-day forecasts from each of our six models were computed for each day in the final eight weeks of each data set, using only data available up through but not including the final eight weeks. This gives us a time series of 40 out-of-sample forecasts from each forecasting method,<sup>10</sup> for each commodity.

One additional forecast was prepared for the out-of-sample testing, which can be viewed as an alternative way of combining market-based forecasts (ISDs) and time series based forecasts (GARCH and HIST). Granger and Ramanathan (1984) argue that if a set of forecasts exists which are either based on different information sets or are based on the same information set but constructed differently, then a better forecast can be obtained by combining the existing forecasts. In our situation, we have forecasts which are constructed from different information sets (e.g., the GARCH forecasts are based on historical information, and the ISD forecasts are based on current market expectations), as well as forecasts constructed from the same information set but constructed differently (e.g., the HIST forecasts and the GARCH forecasts are both based strictly on historical information, but the forecasts are constructed differently). Therefore, combining these forecasts has the potential to generate an improved forecast.

One method of combining these forecasts, suggested by Granger and Ramanathan (1984), is to regress the true volatility on the set of forecasts to obtain weights, then weight all future forecasts by the weights obtained in this preliminary regression. To construct this combined forecast, we

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<sup>10</sup> The GARCH and COMB model parameters are not re-estimated as the 225-day forecast horizon moves through the 40 day window. This biases the results against these two models since their results are only conditioned on the in-sample data and none of the out-of-sample data was used to update the model parameters.

withheld 200 observations from the end of our data sets<sup>11</sup> and reestimated all the models. We then ran the regression

$$(13) \quad \text{ACTUAL}_{t,225} = \gamma_0 + \gamma_1 \text{ISDAT}_{t,225} + \gamma_2 \text{HIST}_{t,225} + \gamma_3 \text{GARCH}_{t,225} + \gamma_4 \text{COMB}_{t,225}$$

to obtain the weights on the forecasts. We did not include ISDAVG and ISD1 in the regression because they are highly collinear with ISDAT (see Table 4), and would therefore add little to forecasting power. Table 6 presents the parameter estimates from (13) for each commodity. Notice that for four of the seven commodities, the COMB forecast gets the highest (positive) weight, suggesting that the GR forecasts are based more heavily on the COMB forecasts than the market-based ISD forecasts. The negative weights which sometimes appear on the GARCH forecasts can be attributed to multicollinearity with the constant. They all become positive when the constant term is omitted from the regression<sup>12</sup>. This linear combination of forecasts is guaranteed to provide superior within-sample forecasts than any of the individual forecasts because it is chosen to minimize within-sample mean squared forecast error. This suggests, but does not guarantee, that it will perform better out-of-sample as well.

TABLE 6

So the final forecast, which we call  $\text{GR}_{t,T}$  (for Granger and Ramanathan), is

$$(14) \quad \text{GR}_{t,225} = \hat{\gamma}_0 + \hat{\gamma}_1 \text{ISDAT}_{t,225} + \hat{\gamma}_2 \text{HIST}_{t,225} + \hat{\gamma}_3 \text{GARCH}_{t,225} + \hat{\gamma}_4 \text{COMB}_{t,225}$$

where the  $\hat{\gamma}_i$ 's come from the in-sample regression (13). See Figure 2 for a graph of the GR forecast for corn. In this figure, the first 406 observations (through February, 1990) were used to construct the GR parameter estimates, meaning that this figure combines both in-sample and out-of-sample forecasts. GR tracks the true volatility very well, increasing when actual volatility increases and decreasing when actual volatility decreases. Unlike the COMB forecast, GR does not overpredict high volatility periods and underpredict low volatility periods.

FIGURE 2

We now have seven forecasts of volatility. The first three (ISDAT, ISDAVG and ISD1) are market-based, the next two (HIST and GARCH) are time series based, and the final two (COMB

<sup>11</sup> In order to compute the true volatility, we need 225 calendar days of returns, which translates into 160 working days, or 160 observations. Therefore, in order to make the Granger and Ramanathan forecasts truly out-of-sample, we need to withhold 160 observations plus the 40 observations from the out-of-sample forecasting period, for 200 observations. Otherwise, the dependent variable in the Granger and Ramanathan regression would include some out-of-sample data.

<sup>12</sup> None of the conclusions of this paper are changed if the constant term is dropped from this regression.

and GR) combine the market and time series based forecasts. The results of the out-of-sample MSFE are presented in Table 7. With the exception of cocoa and silver, the GR forecast has the lowest out-of-sample mean squared forecast error, and for silver the COMB forecast has the lowest. Also, the COMB forecast has the second-lowest MSFE for four of the five commodities where GR has the lowest. The obvious conclusion is that the two combined forecasts perform better than either the time series forecasts or the market based forecasts. This suggests that much more precise interval forecasts can be made using the GR or COMB forecasts of variance. Furthermore, since the GR and COMB forecasts clearly dominate the ISD forecasts, we speculate that the difference between the combined forecasts and the ISD forecasts can be used to identify mispriced options. The idea is that since expectations of future volatility play such a critical role in the determination of options prices, better forecasts of volatility should lead to better pricing and should therefore help an investor identify over- or under-priced options contracts.

TABLE 7

## VI. Conclusions.

The results presented above are promising. In particular, the COMB and GR forecasts, which combine market-based information with time series information, yield better forecasts than can be obtained from market expectations or time series models alone. Several implications of this are immediately apparent. First, the time series contains information about future volatility that is not captured by market expectations, suggesting that options markets are inefficient (and/or the option pricing formula we used is incorrect). This implies that it is possible that our volatility forecast can be used to identify mispriced options, and a profitable trading rule could be established based on the difference between the ISD and the COMB or GR volatility forecast. Second, our forecasting method can be used to obtain interval forecasts of commodity prices, which should be beneficial to market participants who are concerned about the precision of a point forecast. One final note is that the accurate matching of the forecast horizon and the time to maturity of the futures contract is relatively unimportant. Our results indicate that near-maturity options tend to forecast long-run volatility about as well as options that are far from maturity.

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Table 1 Options Data Summary				
Commodity	Dates	# contracts	# days	Average # contracts per day
Cocoa	07/13/87 - 11/21/90	16,033	851	18.8
Cotton	01/05/87 - 11/02/90	27,735	964	28.8
Corn	07/01/88 - 11/28/90	33,349	606	63.3
Gold	01/02/87 - 12/06/90	23,625	987	26.0
Silver	01/02/87 - 11/28/90	26,122	979	26.7
Sugar	01/05/87 - 12/04/90	22,026	982	22.4
Wheat	01/02/87 - 12/10/90	35,859	994	36.1

Table 2 GARCH and COMB Model Estimates								
GARCH: $h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1}$								
COMB: $h_t = \omega + \alpha \epsilon_{t-1}^2 + \beta h_{t-1} + \delta \sigma_{t-1}^2$								
Parameter		Cocoa	Cotton	Corn	Gold	Silver	Sugar	Wheat
GARCH Model	$\omega$	0.069 (2.22)	0.908 (8.79)	0.063 (2.12)	0.062 (4.39)	0.332 (8.75)	0.361 (4.89)	0.799 (7.13)
	$\alpha$	0.101 (6.75)	0.289 (7.02)	0.099 (4.13)	0.062 (4.82)	0.327 (11.86)	0.058 (5.67)	0.206 (5.12)
	$\beta$	0.892 (57.50)	0.074 (0.85)	0.869 (24.95)	0.870 (35.25)	0.615 (28.68)	0.886 (46.36)	0.474 (6.77)
	$Q^2(12)$	10.35	4.12	4.36	4.67	7.14	4.85	0.37
COMB Model	$\omega$	-0.008 (-0.12)	0.383 (4.83)	-0.030 (-1.16)	0.185 (1.58)	-0.269 (-3.30)	2.203 (5.08)	0.603 (3.92)
	$\alpha$	0.141 (6.32)	0.257 (6.61)	0.082 (3.37)	0.027 (1.57)	0.280 (6.36)	0.117 (4.92)	0.275 (4.39)
	$\beta$	0.778 (17.64)	0.038 (0.45)	0.864 (24.60)	-0.369 (-3.02)	0.178 (2.29)	0.020 (0.16)	0.000*
	$\delta$	5.11 (2.49)	14.67 (6.48)	2.307 (3.37)	33.71 (7.20)	25.11 (7.67)	16.76 (5.60)	24.39 (5.43)
	$Q^2(12)$	10.01	5.28	3.78	5.84	4.65	3.61	0.45

\* Due to estimation problems, this parameter was set to zero.

Table 3 Forecast Summary Statistics								
		ACTUAL	ISDAT	ISDAVG	ISD1	HIST	GARCH	COMB
Cocoa	avg	0.358	0.278	0.286	0.281	0.334	0.420	0.367
	min	0.213	0.117	0.117	0.117	0.124	0.342	0.153
	max	0.511	0.546	0.436	0.413	0.654	0.708	0.545
	nobs	777	770	770	765	777	777	777
Cotton	avg	0.185	0.192	0.195	0.194	0.186	0.192	0.189
	min	0.139	0.110	0.116	0.116	0.093	0.192	0.148
	max	0.259	0.412	0.412	0.412	0.384	0.205	0.239
	nobs	890	883	883	774	890	890	890
Corn	avg	0.190	0.232	0.237	0.231	0.208	0.222	0.210
	min	0.136	0.163	0.169	0.130	0.090	0.210	0.085
	max	0.238	0.355	0.316	0.314	0.724	0.297	0.403
	nobs	532	528	528	504	532	532	532
Gold	avg	0.148	0.168	0.171	0.169	0.144	0.154	0.148
	min	0.112	0.129	0.137	0.099	0.081	0.151	0.113
	max	0.215	0.326	0.266	0.267	0.280	0.170	0.275
	nobs	913	906	906	869	913	913	913
Silver	avg	0.253	0.272	0.270	0.270	0.263	0.378	0.274
	min	0.155	0.171	0.185	0.141	0.110	0.368	0.132
	max	0.468	0.604	0.790	0.506	0.738	0.604	0.694
	nobs	905	899	899	816	905	905	905
Sugar	avg	0.391	0.408	0.423	0.405	0.389	0.409	0.402
	min	0.293	0.223	0.253	0.180	0.197	0.400	0.303
	max	0.560	0.915	0.894	0.600	0.865	0.496	0.699
	nobs	908	900	900	812	908	908	908
Wheat	avg	0.239	0.218	0.221	0.219	0.229	0.255	0.255
	min	0.114	0.110	0.110	0.110	0.084	0.254	0.187
	max	0.360	0.478	0.504	0.484	0.593	0.312	0.586
	nobs	920	913	913	913	920	920	920

Table 4 Correlation Matrix for Corn Forecasts						
	ISDAT	ISDAVG	ISD1	HIST	GARCH	COMB
ISDAT	1.000	0.964	0.945	0.167	0.755	0.829
ISDAVG		1.000	0.968	0.155	0.738	0.813
ISD1			1.000	0.128	0.712	0.809
HIST				1.000	0.855	0.313
GARCH					1.000	0.741
COMB						1.000

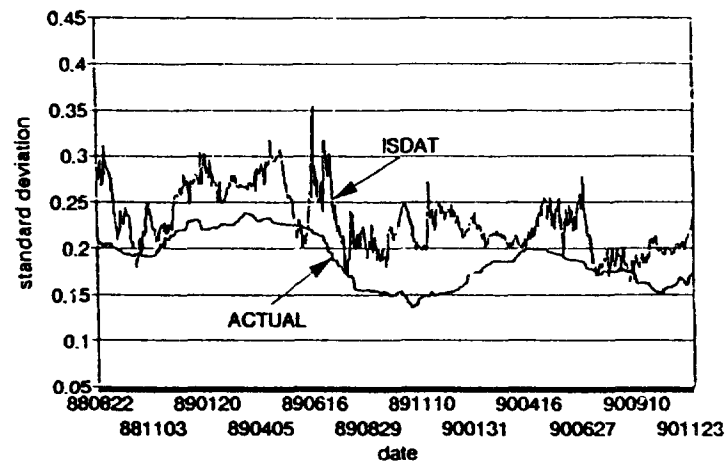
Table 5 Mean Squared Forecast Errors — Full Sample						
	ISDAT	ISDAVG	ISD1	HIST	GARCH	COMB
Cocoa	1.391	1.277	1.322	1.603	1.138	0.938
Cotton	0.152	0.168	0.173	0.382	0.144	0.092
Corn	0.251	0.288	0.264	1.001	0.193	0.271
Gold	0.177	0.193	0.196	0.181	0.086	0.121
Silver	0.615	0.573	0.587	1.161	2.370	0.801
Sugar	1.557	1.548	1.088	1.748	0.537	0.716
Wheat	0.896	0.945	0.916	1.840	0.470	0.763

Table 6					
Granger/Ramanathan OLS Parameter Estimates — Restricted Sample					
$ACTUAL_{t,225} = \gamma_0 + \gamma_1 ISDAT_{t,225} + \gamma_2 HIST_{t,225} + \gamma_3 GARCH_{t,225} + \gamma_4 COMB_{t,225}$					
	constant	ISDAT	HIST	GARCH	COMB
Cocoa	0.181	0.338	0.032	0.349	-0.165
Cotton	-0.154	-0.000	-0.083	0.726	1.106
Corn	0.225	0.333	-0.015	-0.719	0.255
Gold	0.227	-0.170	0.087	-0.922	0.409
Silver	0.056	0.516	0.255	-0.036	-0.022
Sugar	0.319	-0.128	0.023	-0.004	0.310
Wheat	0.173	-0.198	-0.333	0.344	0.430

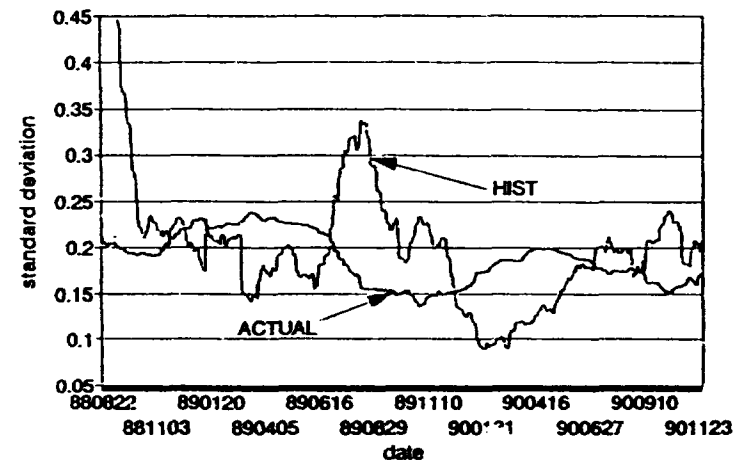
Table 7							
Out of Sample Mean Squared Forecast Errors							
	ISDAT	ISDAVG	ISD1	HIST	GARCH	COMB	GR
Cocoa	0.311	0.300	0.300	1.958	2.788	1.927	1.170
Cotton	0.018	0.020	0.020	0.080	0.091	0.015	0.012
Corn	0.196	0.261	0.209	0.335	0.399	0.090	0.003
Gold	0.227	0.216	0.219	0.414	0.007	0.106	0.001
Silver	0.231	0.235	0.224	0.363	0.831	0.145	0.441
Sugar	1.283	0.716	0.936	0.374	0.295	0.239	0.190
Wheat	0.065	0.065	0.062	0.055	0.120	0.011	0.009



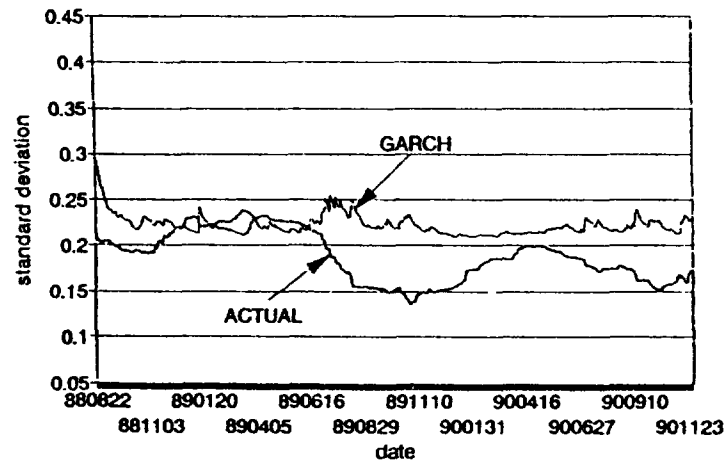
ISDAT Forecasts  
Corn



HIST Forecasts  
Corn



GARCH Forecasts  
Corn



COMB Forecasts  
Corn

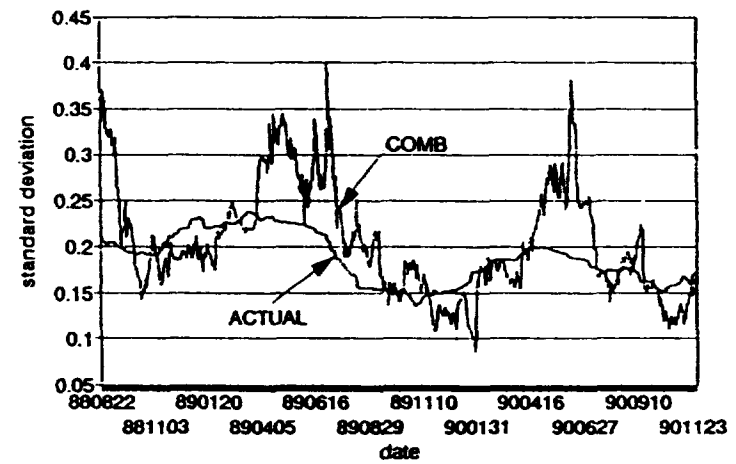


Figure 1

# GR Forecasts

## Corn

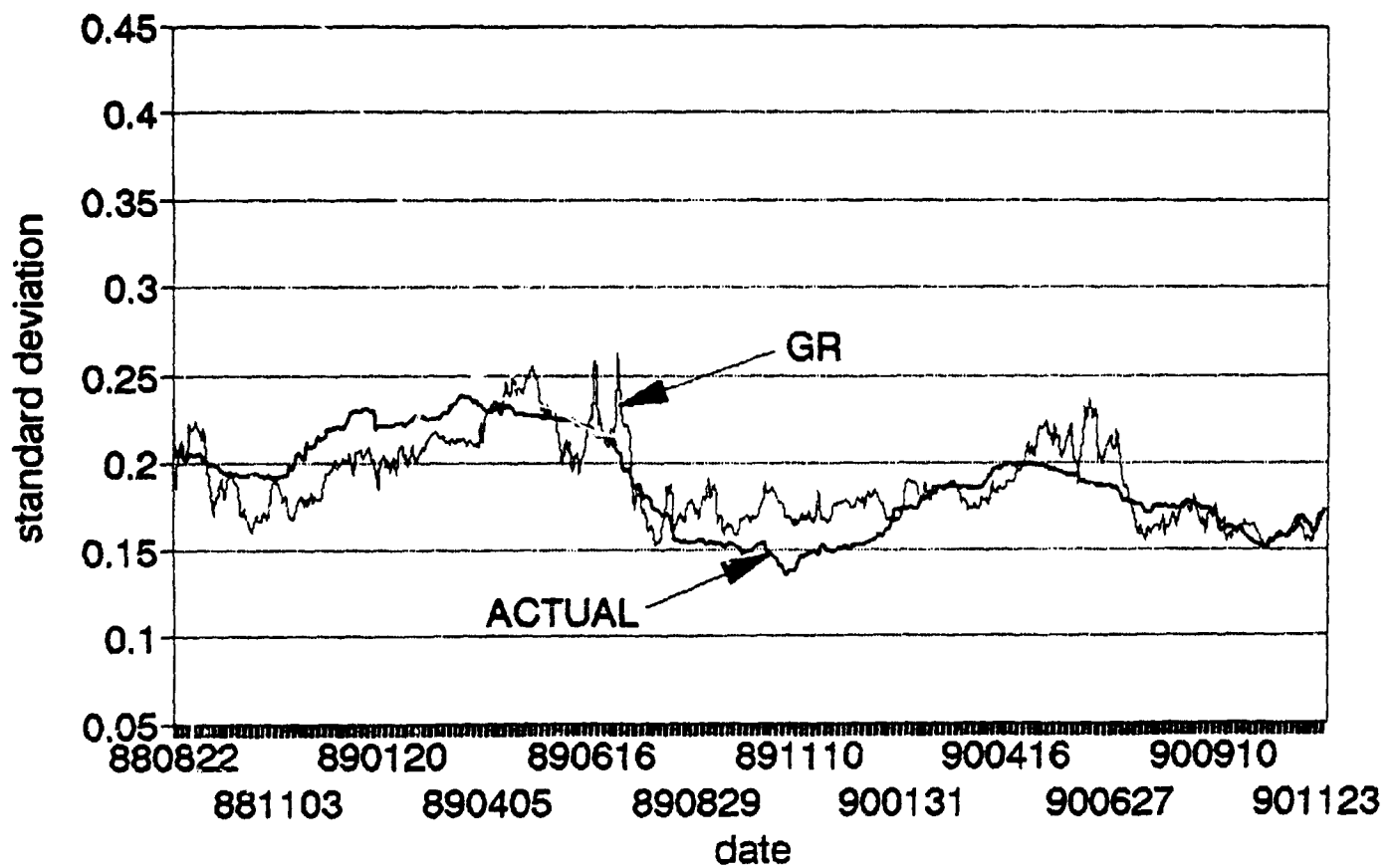


Figure 2

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